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Cabibbo suppressed decays and the Ξ_c^+ lifetime

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Abstract

The problem of the Ξ_c^+ lifetime is considered in the framework of *Heavy-Quark Expansion* and $SU(3)_{flavor}$ symmetry. The lifetime of Ξ_c^+ is expressed in terms of measurable inclusive quantities of the other two charmed baryons belonging to the same $SU(3)_{flavor}$ multiplet in a model-independent way. In such a treatment, inclusive decay rates of singly Cabibbo suppressed decay modes have a prominent role. An analogous approach is applied to the multiplet of charmed mesons yielding interesting predictions on D_s^+ properties. The results obtained indicate that a more precise measurement of inclusive decay quantities of some charmed hadrons (such as Λ_c^+) that are more amenable to experiment can contribute significantly to our understanding of decay properties of other charmed hadrons (such as Ξ_c^+) where discrepancies or ambiguities exist.

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The investigation of inclusive decays and lifetimes of hadrons containing heavy quarks [1] is already a mature subject with many fruitful applications and numerous significant achievements. The fusion of the *Operator Product Expansion (OPE)* techniques developed in the nineties [2] with the phenomenological insights of the eighties [3] has created a consistent framework known as *Heavy-Quark Expansion (HQE)*, within which one can systematically treat inclusive decays of heavy quarks and hadrons containing them. A host of experimental data, first on c hadron decays and then, with the advent of B factories, on b decays, have made possible a comparison of experimental and theoretical results and revealed broad agreement with several notable exceptions [1]. Addressing these discrepancies has become one of the most important tasks in heavy-quark physics, given that data extracted from inclusive weak decays represent an essential input in

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[1]Like the still present problem of the $\tau(\Lambda_b^0)/\tau(B_d^0)$ ratio or the recently escalating problem of the $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ ratio.

research of fundamental questions of the Standard Model (such as *CP* violation) or its extensions. Increasing quantity and quality of experimental data opens new directions in treating inclusive weak decays which may contribute to the resolution of existing problems. Consideration of inclusive weak decay rates of Cabibbo suppressed modes as individual objects (not only as a small correction to inclusive weak decay rates of Cabibbo dominant modes) supported by the application of standard symmetries (such as $SU(3)_{flavor}$ or *Heavy-Quark Symmetry (HQS)*) traces along one of these directions.

As the *HQE* depends crucially on the heaviness of the decaying heavy quark, the predictions are more reliable in the sector of b hadrons than in the sector of c hadrons. Nevertheless, reliable predictions of lifetime hierarchies and lifetime ratios were obtained in the sector of charmed hadrons too. Furthermore, very good quantitative agreement was achieved in the sector of singly-charmed baryons [4, 5]. However, recent measurements of the Ξ_c^+ lifetime by *FOCUS* [6] and *CLEO* [7] collaborations indicate substantial discrepancy between new experimental data and the presently available theoretical result [4, 8]:

$$\begin{aligned} \left(\tau(\Xi_c^+)/\tau(\Lambda_c^+)\right)_{FOCUS} &= 2.29 \pm 0.14\,, \\ \left(\tau(\Xi_c^+)/\tau(\Lambda_c^+)\right)_{CLEO} &= 2.8 \pm 0.3\,, \\ \left(\tau(\Xi_c^+)/\tau(\Lambda_c^+)\right)_{th} &\sim 1.3\,. \end{aligned} \tag{1}$$

The results displayed above show that there is a difference by a factor of ~ 2 between experiment and theory. It is reasonable to pose a question whether the *HQE* can correctly describe lifetimes of singly-charmed baryons. The new experimental data on the lifetime of Ξ_c^+ are certainly out of reach of the calculations performed so far. However, experimental data for other singly-charmed, weakly-decaying baryons (Λ_c^+, Ξ_c^0, and Ω_c^0) are consistent with theoretical calculations of [4]. The theoretical procedure is based on some assumptions (e.g., calculation of four-quark operator matrix elements in a nonrelativistic quark model) that may limit its explanatory power in the case of Ξ_c^+. Therefore, it is justified to investigate if a theoretical procedure based on the *HQE* with relaxed assumptions of analysis [4] can be formulated so that it might explain new experimental results. To this end, one must invoke Cabibbo suppressed modes of decay as a new source of information.

Let us begin our analysis with a brief discussion about the inclusive weak decay rate. The principal result of the *HQE* is the expression for any inclusive weak decay rate of a heavy hadron given as a series of matrix elements of local operators with the inverse mass of the decaying heavy quark as an expansion parameter:

$$\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} \mid V \mid^2 \frac{1}{2M_{H_Q}} \times \left[\sum_{D=3}^{\infty} c_D^f \frac{\langle H_Q \mid O_D \mid H_Q \rangle}{m_Q^{D-3}} \right], \tag{2}$$

where D denotes the canonical dimension of the local operator O_D. The coefficients c_D are calculated perturbatively (therefore given as a series in α_S). V stands for a product of *CKM* matrix elements appearing in a given weak decay mode. For the sake of practical calculations, one has to truncate the series at some point in the series hoping that the disregarded remainder of the series does not contribute significantly to the final result. The quality of such an approximation depends on the magnitude of the expansion parameter, i.e., on the speed of convergence of the series. The underlying hypothesis is that the inclusive hadron decay rates can be described by calculating the inclusive quark decay rates – the *ansatz* known as quark-hadron duality. The *ansatz* is not trivially obvious as one can see by inspection of the leading term in (2). The decay rate is given by $\Gamma^{dec} \sim m_Q^5$ and this expression has, *prima facie*, nothing to do with the hadrons in the final states. This is, however, misleading since the summation of hadronic widths of different channels agrees with the widths computed at the quark level [2]. Another problem stems from the matrix elements appearing in the expansion. They are dominated by nonperturbative dynamics and therefore so far there has been no systematic way of calculating them. The matrix elements of several operators of the lowest dimensions can be determined applying *Heavy-Quark Effective Theory (HQET)*, but the matrix elements of some operators essential for understanding lifetime differences of heavy hadrons (e.g., four-quark operators) are not calculable in such a manner, but one must recourse to quark models, which introduces the undesirable feature of model dependence. A further source of uncertainty is the heavy-quark mass m_Q. Since in the leading order the inclusive weak decay rate depends on the fifth power of m_Q, very small uncertainties in the determination of this mass parameter can lead to a significant uncertainty in the inclusive weak decay rate. Finally, using a truncated expression instead of the entire series raises the possibility of violation of quark-hadron duality [11, 12], which emerges as another possible source of contributions beyond the present theoretical control.

The *OPE* was originally formulated for deep Euclidean kinematics and its net effect was to factorize perturbative short-distance physics (Wilson coefficients) from soft, nonperturbative one (matrix elements). On the other hand, the quark-hadron duality is the concept dealing exclusively with Minkowskian dynamics [3]. It appears that the small corrections that one safely neglects in the Euclidean regime often turn out to be enhanced in the Minkowski regime [11, 12]. The Wilson coefficients themselves are generally not free of nonperturbative (nonlogarithmic) terms. They are generated, e.g., by small-size instantons [11]. Similarly, perturbative corrections appear in the soft physics of matrix elements. Generally, the truncation of the series (2) in α_s and condensate terms is known to be necessary since both series are factorially divergent [13]. Therefore, a "practical" calculation at any given order α_s^m and m_Q^{-n} will have a "natural uncertainty" coming from the higher-order terms α_s^{m+1} and $m_Q^{-(n+1)}$. The "natural uncertainty" also includes the ordinary uncertainties like the uncertainties in quark masses, Λ_{QCD}, etc. The uncertainties beyond this "natural uncertainty" are considered to be violations of quark-hadron duality.

Resolutions of the problems stated above presumably lead to the explanation of discrepancies between present experimental and theoretical results. Since the contributions of higher-dimensional operators, uncertainties in matrix elements and m_Q as well as effects of duality violation are all intertwined in the full expression for the weak decay rate, it is very difficult to distinguish precisely which of these factors causes the problem and should be improved accordingly. One possible strategy is to eliminate or reduce the importance of all (in practice as many as possible) factors but one in order to test the influence of the remaining factor. In this paper we adopt this strategy and implement it using symmetries in multiplets of heavy hadrons. Investigations along similar lines (connecting the charmed with the beauty sector) were performed in [14, 15, 16].

[2] As nicely demonstrated in $(1+1)$-dimensional *QCD* [9, 10, 11].

[3] Therefore, it cannot be studied in lattice QCD, which is essentially a numerical Euclidean approach.

The standard procedure of truncating the series (2) is to keep operators of dimensions 3 (decay operator) and 5 (chromomagnetic operator) [4], which are insensitive to the light-quark content of the heavy hadron (at least on the quark-gluon operator level). Operators of dimension 6, which are sensitive to flavors of light quarks (four-quark operators), also have to be kept in order to describe lifetime differences within multiplets of heavy hadrons. The effects of four-quark operators (clearly presented in [4]) are traditionally referred to as W-exchange, positive and negative Pauli interference in baryons, and W-annihilation, W-exchange, and negative Pauli interference in mesons. We shall adopt this procedure along with the assumption of $SU(3)_{flavor}$ symmetry at the level of matrix elements. The validity of this assumption and its influence on the final result will be discussed below.

We start by expressing decay rates of individual Cabibbo modes for singly-charmed baryons within the framework that we have set. As already mentioned, operators of dimension 3 and 5 are insensitive to the light-(anti)quark content of a heavy hadron. Nevertheless, their coefficients comprise a phase-space correction coming from the fact that some of the resulting quarks in the decay of a heavy quark have a nonnegligible mass compared with the heavy-quark mass. Thus, contributions of operators of dimensions 3 and 5 have slightly different values in the treatment of various Cabibbo modes of the decay of the heavy quark. In the case of c quark decays, these corrections are generally not large and we shall neglect them in our initial treatment. Their effect will be taken into account in the discussion of our results. The assumption of $SU(3)_{flavor}$ symmetry guarantees that the matrix elements of operators of dimension 3 and 5 are the same for all hadrons in any $SU(3)_{flavor}$ multiplet of heavy hadrons. These approximations allow us to describe the contribution of the aforementioned operators with a single quantity Γ_{35} in all Cabibbo modes, for all members of the multiplet, up to the product of the CKM matrix elements specific for every individual Cabibbo mode. The coefficients of four-quark operators also include phase-space corrections owing to the massive particles in the final state of the decay of the heavy quark. In this case, however, these corrections are at the percent level and can be safely disregarded in c quark decays. The contributions of these operators of dimension 6 for the case of baryons can then be expressed in terms of several parameters (under the assumption of $SU(3)_{flavor}$ symmetry) related to the aforementioned types of four-quark effects: W-exchange (Γ_{exch}), negative Pauli interference (Γ_{negint}), and positive Pauli interference (Γ_{posint}), again up to the CKM matrix elements. Analogous claims are valid in the case of charmed meson decays. We should emphasize that Γ's are conveniently chosen combinations of products of coefficients and operator matrix elements which appear in expressions for the inclusive weak decay rates of all Cabibbo modes. As we do not engage in a direct calculation of matrix elements, all these matrix elements can be taken as determined at the same scale μ, i.e., there is no need for the hybrid renormalization in the case of four-quark operators. One needs to know nothing else about the matrix elements of the operators. In such a suitably defined theoretical environment one can express inclusive decay rates in a straightforward manner. The decay rates for nonleptonic modes are

$$\frac{\Gamma^{c\to s\bar{d}u}(\Lambda_c^+)}{|V_{cs}|^2|V_{ud}|^2} = \Gamma_{35} + \Gamma_{exch} + \Gamma_{negint} \, ,$$

[4]there are no operators of dimension 4 owing to color-gauge invariance

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{s}u}(\Lambda_c^+)}{|V_{cs}|^2|V_{us}|^2} &= \Gamma_{35}+\Gamma_{negint}\,, \\
\frac{\Gamma^{c\to d\bar{d}u}(\Lambda_c^+)}{|V_{cd}|^2|V_{ud}|^2} &= \Gamma_{35}+\Gamma_{exch}+\Gamma_{negint}+\Gamma_{posint}\,, \\
\frac{\Gamma^{c\to d\bar{s}u}(\Lambda_c^+)}{|V_{cd}|^2|V_{us}|^2} &= \Gamma_{35}+\Gamma_{posint}+\Gamma_{negint}
\end{aligned} \tag{3}$$

for Λ_c^+,

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{d}u}(\Xi_c^+)}{|V_{cs}|^2|V_{ud}|^2} &= \Gamma_{35}+\Gamma_{posint}+\Gamma_{negint}\,, \\
\frac{\Gamma^{c\to s\bar{s}u}(\Xi_c^+)}{|V_{cs}|^2|V_{us}|^2} &= \Gamma_{35}+\Gamma_{negint}+\Gamma_{posint}+\Gamma_{exch}\,, \\
\frac{\Gamma^{c\to d\bar{d}u}(\Xi_c^+)}{|V_{cd}|^2|V_{ud}|^2} &= \Gamma_{35}+\Gamma_{negint}\,, \\
\frac{\Gamma^{c\to d\bar{s}u}(\Xi_c^+)}{|V_{cd}|^2|V_{us}|^2} &= \Gamma_{35}+\Gamma_{exch}+\Gamma_{negint}
\end{aligned} \tag{4}$$

for Ξ_c^+, and

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{d}u}(\Xi_c^0)}{|V_{cs}|^2|V_{ud}|^2} &= \Gamma_{35}+\Gamma_{posint}+\Gamma_{exch}\,, \\
\frac{\Gamma^{c\to s\bar{s}u}(\Xi_c^0)}{|V_{cs}|^2|V_{us}|^2} &= \Gamma_{35}+\Gamma_{posint}+\Gamma_{exch}\,, \\
\frac{\Gamma^{c\to d\bar{d}u}(\Xi_c^0)}{|V_{cd}|^2|V_{ud}|^2} &= \Gamma_{35}+\Gamma_{posint}+\Gamma_{exch}\,, \\
\frac{\Gamma^{c\to d\bar{s}u}(\Xi_c^0)}{|V_{cd}|^2|V_{us}|^2} &= \Gamma_{35}+\Gamma_{posint}+\Gamma_{exch}
\end{aligned} \tag{5}$$

for Ξ_c^0. For the decay rates of the semileptonic modes one obtains ($l = e, \mu$)

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{l}\nu_l}(\Lambda_c^+)}{|V_{cs}|^2} &= \Gamma_{35}^{SL} \\
\frac{\Gamma^{c\to d\bar{l}\nu_l}(\Lambda_c^+)}{|V_{cd}|^2} &= \Gamma_{35}^{SL}+\Gamma_{posint}^{SL}
\end{aligned} \tag{6}$$

for Λ_c^+,

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{l}\nu_l}(\Xi_c^+)}{|V_{cs}|^2} &= \Gamma_{35}^{SL}+\Gamma_{posint}^{SL} \\
\frac{\Gamma^{c\to d\bar{l}\nu_l}(\Xi_c^+)}{|V_{cd}|^2} &= \Gamma_{35}^{SL}
\end{aligned} \tag{7}$$

for Ξ_c^+, and

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{l}\nu_l}(\Xi_c^0)}{|V_{cs}|^2} &= \Gamma_{35}^{SL} + \Gamma_{posint}^{SL} \\
\frac{\Gamma^{c\to d\bar{l}\nu_l}(\Xi_c^0)}{|V_{cd}|^2} &= \Gamma_{35}^{SL} + \Gamma_{posint}^{SL}
\end{aligned} \tag{8}$$

for Ξ_c^0. One can introduce the following notation for the *CKM* matrix elements: $|V_{cs}|^2 = |V_{ud}|^2 = (\cos\theta_c)^2 \equiv c^2$ and $|V_{cd}|^2 = |V_{us}|^2 = (\sin\theta_c)^2 \equiv s^2$. Combining relations from (3) and (4), one obtains

$$\begin{aligned}
\Gamma^{c\to s\bar{d}u}(\Xi_c^+) &= \frac{c^2}{s^2}\left(\Gamma^{c\to s\bar{s}u}(\Lambda_c^+) + \Gamma^{c\to d\bar{d}u}(\Lambda_c^+)\right) - \Gamma^{c\to s\bar{d}u}(\Lambda_c^+)\,, \\
\Gamma^{c\to s\bar{s}u}(\Xi_c^+) &+ \Gamma^{c\to d\bar{d}u}(\Xi_c^+) = \Gamma^{c\to s\bar{s}u}(\Lambda_c^+) + \Gamma^{c\to d\bar{d}u}(\Lambda_c^+)\,, \\
\Gamma^{c\to d\bar{s}u}(\Xi_c^+) &= \frac{s^4}{c^4}\Gamma^{c\to s\bar{d}u}(\Lambda_c^+)
\end{aligned} \tag{9}$$

for the nonleptonic decay rates and from (6), (7), and (8) we have

$$\Gamma_{SL}(\Xi_c^+) = \Gamma_{SL}(\Xi_c^0) + \frac{s^2}{c^2}(\Gamma_{SL}(\Lambda_c^+) - \Gamma_{SL}(\Xi_c^0)) \tag{10}$$

for the semileptonic decay rates, where $\Gamma_{SL}(X) = \Gamma^{c\to s\bar{l}\nu_l}(X) + \Gamma^{c\to d\bar{l}\nu_l}(X)$, $X = \Xi_c^+, \Xi_c^0, \Lambda_c^+$. Expressions (9) and (10) show that all contributions to the total inclusive weak decay rate of Ξ_c^+ are expressed in terms of some of the analogous contributions of Λ_c^+ and Ξ_c^0. In this way, we have succeeded in expressing the lifetime of a "problematic" baryon Ξ_c^+ in terms of quantities of "nonproblematic" baryons Λ_c^+ and Ξ_c^0. If we introduce the notation

$$BR_{\Delta C=-1,\Delta S=0}(\Lambda_c^+) = \left(\Gamma^{c\to s\bar{s}u}(\Lambda_c^+) + \Gamma^{c\to d\bar{d}u}(\Lambda_c^+)\right)/\Gamma_{TOT}(\Lambda_c^+)\,, \tag{11}$$

the final expression (after neglecting all terms $\sim s^4$) for the ratio specified in (1) becomes

$$\begin{aligned}
\frac{\tau(\Xi_c^+)}{\tau(\Lambda_c^+)} = &\left[-1 + \left(2 + \frac{c^2}{s^2}\right) BR_{\Delta C=-1,\Delta S=0}(\Lambda_c^+)\right. \\
&\left. + 2\left(1 - \frac{s^2}{c^2}\right)\frac{\tau(\Lambda_c^+)}{\tau(\Xi_c^0)} BR_{SL}(\Xi_c^0) + 2\left(1 + \frac{s^2}{c^2}\right) BR_{SL}(\Lambda_c^+)\right]^{-1}\,.
\end{aligned} \tag{12}$$

This type of analysis can be extended to the sector of charmed mesons. The hierarchy of charmed meson lifetimes is in general well understood in the framework of the *HQE* [17], although some disrepancies exist that motivate alternative approaches [18] and raise corresponding controversies [19]. We shall pursue our analysis in the framework of *HQS* and perform a model-independent analysis. This analysis, apart from its intrinsic value as a contribution to the understanding of charmed meson lifetimes, can also be a testing ground of our approach because of a higher quality of available experimental data for charmed mesons. Therefore, we conduct our analysis on a $SU(3)_{flavor}$ antitriplet of charmed mesons. The inclusive weak decay rates for individual Cabibbo nonleptonic decay modes are (Γ's used in the mesonic case are different from those used in the baryonic case although the notation is very similar)

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{d}u}(D^+)}{|V_{cs}|^2|V_{ud}|^2} &= \Gamma_{35} + \Gamma_{negint}\,, \\
\frac{\Gamma^{c\to s\bar{s}u}(D^+)}{|V_{cs}|^2|V_{us}|^2} &= \Gamma_{35}\,, \\
\frac{\Gamma^{c\to d\bar{d}u}(D^+)}{|V_{cd}|^2|V_{ud}|^2} &= \Gamma_{35} + \Gamma_{ann} + \Gamma_{negint}\,, \\
\frac{\Gamma^{c\to d\bar{s}u}(D^+)}{|V_{cd}|^2|V_{us}|^2} &= \Gamma_{35} + \Gamma_{ann}
\end{aligned} \tag{13}$$

for D^+,

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{d}u}(D^0)}{|V_{cs}|^2|V_{ud}|^2} &= \Gamma_{35} + \Gamma_{exch}\,, \\
\frac{\Gamma^{c\to s\bar{s}u}(D^0)}{|V_{cs}|^2|V_{us}|^2} &= \Gamma_{35} + \Gamma_{exch}\,, \\
\frac{\Gamma^{c\to d\bar{d}u}(D^0)}{|V_{cd}|^2|V_{ud}|^2} &= \Gamma_{35} + \Gamma_{exch}\,, \\
\frac{\Gamma^{c\to d\bar{s}u}(D^0)}{|V_{cd}|^2|V_{us}|^2} &= \Gamma_{35} + \Gamma_{exch}
\end{aligned} \tag{14}$$

for D^0, and

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{d}u}(D_s^+)}{|V_{cs}|^2|V_{ud}|^2} &= \Gamma_{35} + \Gamma_{ann}\,, \\
\frac{\Gamma^{c\to s\bar{s}u}(D_s^+)}{|V_{cs}|^2|V_{us}|^2} &= \Gamma_{35} + \Gamma_{ann} + \Gamma_{negint}\,, \\
\frac{\Gamma^{c\to d\bar{d}u}(D_s^+)}{|V_{cd}|^2|V_{ud}|^2} &= \Gamma_{35}\,, \\
\frac{\Gamma^{c\to d\bar{s}u}(D_s^+)}{|V_{cd}|^2|V_{us}|^2} &= \Gamma_{35} + \Gamma_{negint}
\end{aligned} \tag{15}$$

for D_s^+. For the decay rates of the semileptonic modes one obtains ($l = e, \mu$)

$$\frac{\Gamma^{c\to s\bar{l}\nu_l}(D^+)}{|V_{cs}|^2} = \Gamma_{35}^{SL}$$

$$\frac{\Gamma^{c\to d\bar{l}\nu_l}(D^+)}{|V_{cd}|^2} = \Gamma_{35}^{SL} + \Gamma_{ann}^{SL} \tag{16}$$

for D^+,

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{l}\nu_l}(D^0)}{|V_{cs}|^2} &= \Gamma_{35}^{SL} \\
\frac{\Gamma^{c\to d\bar{l}\nu_l}(D^0)}{|V_{cd}|^2} &= \Gamma_{35}^{SL}
\end{aligned} \tag{17}$$

for D^0, and

$$\begin{aligned}
\frac{\Gamma^{c\to s\bar{l}\nu_l}(D_s^+)}{|V_{cs}|^2} &= \Gamma_{35}^{SL} + \Gamma_{ann}^{SL} \\
\frac{\Gamma^{c\to d\bar{l}\nu_l}(D_s^+)}{|V_{cd}|^2} &= \Gamma_{35}^{SL}
\end{aligned} \tag{18}$$

for D_s^0. Combining relations from (13) and (15), one obtains

$$\begin{aligned}
\Gamma^{c\to s\bar{d}u}(D_s^+) &= \frac{c^2}{s^2}\left(\Gamma^{c\to s\bar{s}u}(D^+) + \Gamma^{c\to d\bar{d}u}(D^+)\right) - \Gamma^{c\to s\bar{d}u}(D^+)\,, \\
\Gamma^{c\to s\bar{s}u}(D_s^+) &+ \Gamma^{c\to d\bar{d}u}(D_s^+) = \Gamma^{c\to s\bar{s}u}(D^+) + \Gamma^{c\to d\bar{d}u}(D^+)\,, \\
\Gamma^{c\to d\bar{s}u}(D_s^+) &= \frac{s^4}{c^4}\Gamma^{c\to s\bar{d}u}(D^+)
\end{aligned} \tag{19}$$

for the nonleptonic decay rates and from (16), (17) and (18) we have

$$\Gamma_{SL}(D_s^+) = \Gamma_{SL}(D^0) + \frac{c^2}{s^2}(\Gamma_{SL}(D^+) - \Gamma_{SL}(D^0)) \tag{20}$$

for the semileptonic decay rates, where $\Gamma_{SL}(X) = \Gamma^{c\to s\bar{l}\nu_l}(X) + \Gamma^{c\to d\bar{l}\nu_l}(X)$, $X = D^+, D^0, D_s^+$. Expressions (19) and (20) show that all contributions to the total inclusive weak decay rate of D_s^+ are expressed in terms of some of the analogous contributions of D^+ and D^0. Let us comment briefly on the findings of [20, 21, 22] which indicate that the *HQE* could not reproduce semileptonic inclusive widths of charmed mesons. Let us point out that although the expressions for semileptonic inclusive decay widths are calculated using the *HQE*, the relations among them (such as (20)) simply state that inclusive semileptonic widths for all three charmed mesons are very close, which is satisfied very well experimentally [5]. Therefore, the possibility that the *HQE* does not describe semileptonic inclusive widths ideally (although contributions of higher dimensional operators should be investigated before making this statement definite) does not bare a consequence on our final results which depend only on the relations among semileptonic decay widths. If we introduce the notation

$$BR_{\Delta C=-1,\Delta S=0}(D^+) = \left(\Gamma^{c\to s\bar{s}u}(D^+) + \Gamma^{c\to d\bar{d}u}(D^+)\right)/\Gamma_{TOT}(D^+)\,, \tag{21}$$

we obtain the following final expression (after neglecting terms $\sim s^4$) for the ratio of lifetimes of D^+ and D^0 mesons

$$\begin{aligned} \frac{\tau(D^+)}{\tau(D_s^+)}(1 - BR_\tau(D_s^+)) &= -1 + \left(2 + \frac{c^2}{s^2}\right) BR_{\Delta C=-1,\Delta S=0}(D^+) \\ &\quad +2\left(1 - \frac{c^2}{s^2}\right)\frac{\tau(D^+)}{\tau(D^0)} BR_{SL}(D^0) + 2\left(1 + \frac{c^2}{s^2}\right) BR_{SL}(D^+)\,, \end{aligned} \tag{22}$$

where $BR_\tau(D_s^+)$ denotes the branching ratio of the leptonic $D_s^+ \to \tau^+\nu_\tau$ decay [5].

Once we have obtained the results (12) and (22), we can clearly see their theoretical and experimental appeal. These relations have an intrinsic value since they express the lifetime of one charmed hadron in terms of measurable quantities of other two charmed hadrons belonging to the same $SU(3)_{flavor}$ multiplet. This result represents exploitation of advantages of the HQE at a new deeper level. The approach that leads to (12) and (22) also suppresses some of the problems referred to in the introduction. Let us briefly discuss these problems in the light of our approach.

The problem of convergence seems rather important in charmed baryon decays. The operators of the lowest dimension in (2), which are neglected in our approach, are some operators of dimension 6 (which are insensitive to the light content of the heavy hadron) followed by the operators of dimension 7 and higher. In our approach, all operators that are insensitive to the light content of heavy hadrons give the same contribution to the inclusive weak decay rate of each Cabibbo mode (up to the CKM matrix elements) and for every hadron within a given $SU(3)_{flavor}$ multiplet. If we look at the relations (9), (10), (19), and (20) as relations between exact inclusive weak decay rates for individual Cabibbo modes (and not only as approximations with several lowest dimensional operators), we can see that contributions of all light-flavor insensitive operators (of any dimension) get cancelled. Thus, these relations are correct up to the contributions of higher light-flavor sensitive operators. Since apart from four-quark operators there are other operators of dimension 6 in (2) but they are all light-flavor insensitive, the aforementioned relations get the first correction from those operators of dimension 7 (or higher) which are light-flavor sensitive. Therefore, relations (9), (10), (19), and (20) are in the form that ameliorates the convergence issue.

The phase-space corrections represent corrections which are different in various Cabibbo modes, depending on the number of massive quarks in the final state. Still, relations (9), (10), (19), and (20) are in such a form that the effect of phase space is significantly reduced. Let us consider the first equation of (9): the sum of decay rates of two modes with one s quark in the final state equals (up to the CKM matrix elements) the sum of decay rates of modes with two and zero s quarks in the final state. Numerical values of the phase-space corrections to operators of dimensions 3 and 5 [17] indicate that the sum of corrections for two s quarks and zero s quarks in the final state is very close to the double correction for one s quark in the final state. The effects

[5] This mode contributes significantly only to the decays of the D_s^+ meson and therefore cannot be related to the analogous decay rates of other members of the $SU(3)_{flavor}$ multiplet.

of phase-space corrections largely cancel. A similar situation appears in all other relations in (9), (10), (19), and (20). Therefore, inclusion of phase-space corrections does not notably worsen the accuracy of the aforementioned relations.

The problem of calculating matrix elements is in our approach completely avoided. From the span of lifetimes of charmed hadrons [5] it is clear that four-quark operators must play a very prominent role. Since, in contradistinction to operators of dimension 3 and 5, the matrix elements of four-quark operators cannot be calculated in a model-independent way, it is clear that even a modest inaccuracy in their determination may lead to significant deviations from the correct result. Moreover, a recent analysis [23] indicates that there might exist serious deviations from some standard approximations, such as the valence quark approximation. Evading these pitfalls is one of the greatest advantages of our approach.

Another advantage is that all crucial relations in this paper do not depend on the heavy quark mass m_Q in the case when the assumed symmetries apply. In the realistic case, the form of relations (9), (10), (19), and (20) reduces the dependence of results on m_Q significantly (to the level of breaking of underlying symmetries).

Finally, there remains the assumption on $SU(3)_{flavor}$ symmetry. The effects of breaking of this symmetry were analyzed in [4]. From that analysis one can conclude that the effects of $SU(3)_{flavor}$ breaking are generally less than 30% and probably significantly smaller. Therefore, we expect the same level of accuracy in our treatment, too.

After the discussion of theoretical advantages and limitations of our approach there remains an important problem of confrontation of theoretical findings with experimental values. From the final relation for baryons (12) and mesons (22) it is evident that theoretical predictions depend on the branching ratios of the singly Cabibbo suppressed nonleptonic modes $BR_{\Delta C=-1,\Delta S=0}(\Lambda_c^+)$ and $BR_{\Delta C=-1,\Delta S=0}(D^+)$, respectively. These values are not available from experiment and their determination represents a crucial step in numerical analysis. An estimate of these quantities can be obtained indirectly from exclusive modes and depends on the quality of data for these modes. From the flavor quantum numbers of the final decay products in heavy-hadron decays one can determine which Cabibbo mode governed that particular decay at the quark level. The only exceptions are the modes $c \to s\bar{s}u$ and $c \to d\bar{d}u$ which lead to the final hadronic state with the same flavor quantum numbers. However, this fact does not pose a problem since in all expressions the decay rates of these two modes appear in the form of sum and therefore there is no need to make difference between them. From the flavor quantum numbers of the final states of any particular exclusive mode one can determine whether it was governed by the Cabibbo dominant, singly Cabibbo suppressed, or doubly Cabibbo suppressed nonleptonic modes at the quark level. An analogous conclusion follows for semileptonic decays. It is, therefore, possible to obtain a decay rate for any Cabibbo inclusive mode (all decay channels coming from the same Cabibbo mode at the quark level) by summing the decay rates of associated exclusive modes. In performing this procedure one encounters the effect of quantum interference. Namely, different final states originating from the same quark decay mode can mix owing to final state strong interactions. The most notable manifestation of this effect is that summing of the branching ratios of all exclusive modes taken from [5] can lead to a result well over 100% (e.g., for D^0 or D^+). To minimize this effect, we invoke the following procedure: we calculate the inclusive decay rate of singly Cabibbo suppressed modes by summing the decay rates of all appropriate exclusive decay modes; then we calculate the *total decay rate* by summing decay rates of *all* exclusive modes and then divide the two numbers to obtain the wanted ratio. Using the sum of all exclusive modes instead of the measured lifetime for the total decay rate insures the same treatment of interference effects in both quantities in the ratio.

Other quantities appearing in the expressions (12) and (22) are lifetimes and semileptonic branching ratios, which are a standard part of information on any weakly decaying particle. In general, they are well measured and available in [5].

Let us first consider the presently very interesting question of the $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ ratio. The sum of branching ratios of all measured exclusive decay modes is approximately 50% which shows that the set of available decay modes is not complete. The branching ratio $BR_{\Delta C=-1,\Delta S=0}(\Lambda_c^+)$ is obtained at the level of 0.0295 ± 0.0115, which is probably an underestimated result since only a few exclusive modes corresponding to singly Cabibbo suppressed modes are available [5]. Another problem is the lack of data on the semileptonic branching ratio of the Ξ_c^0 baryon. This value can be taken from [4] to be $BR_{SL}(\Xi_c^0) = (0.092 \pm 0.006)$. As the contribution coming from $BR_{SL}(\Xi_c^0)$ is the nonleading one (the leading one coming from the $BR_{\Delta C=-1,\Delta S=0}(\Lambda_c^+)$), this mixing of theoretical and experimental results does not introduce a significant model dependence. Still, only the arrival of experimental data on $BR_{SL}(\Xi_c^0)$ (hopefully in the near future) will complete the set of experimental values needed for a fully consistent analysis. The rest of the data is taken to be [5]: $BR_{SL}(\Lambda_c^+) = (0.045 \pm 0.017)$, $\tau(\Lambda_c^+) = (0.206 \pm 0.012)\,\mathrm{ps}$, and $\tau(\Xi_c^0) = (0.098 \pm 0.019)\,\mathrm{ps}$. The analysis using the set of parameters specified above yields a result for the $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ ratio which is far above the new experimental results and has a very large error. The principal reason for such a result can be seen from (12). The value of $BR_{\Delta C=-1,\Delta S=0}(\Lambda_c^+)$ is multiplied by a large factor c^2/s^2, which makes the final result very sensitive to the value of this branching ratio. The conclusion stemming from this analysis is that the presently available data on Λ_c^+ exclusive modes are insufficiently accurate and abundant to insure a reliable result. A more extensive and precise measurement of exclusive decay modes of Λ_c^+ (especially Cabibbo suppressed ones) can however lead to interesting new predictions on Ξ_c^+.

Numerical analysis in the sector of charmed meson decays is more promising. Addition of the branching ratios of all exclusive modes of D^+ gives a value of 110%, which shows that the data on exclusive decay modes of D^+ can be considered complete. The branching ratio $BR_{\Delta C=-1,\Delta S=0}(D^+)$ attains the value 0.140 ± 0.026. Other values taken from [5] are $BR_{SL}(D^+) = 0.172 \pm 0.019$, $BR_{SL}(D^0) = 0.0675 \pm 0.0029$, $\tau(D^+) = (1.051 \pm 0.013)\,\mathrm{ps}$, and $\tau(D^0) = (0.4126 \pm 0.0028)\,\mathrm{ps}$. Using the expression (22) one obtains the value $(\tau(D^+)/\tau(D_s^+))(1-BR_\tau(D_s^+))_{th} = 2.63 \pm 0.98$. This result obtained from theoretical considerations can be compared with the value for the same quantity following from the experiment. To this end, we use the experimental values [5]: $\tau(D_s^+) = (0.496 \pm 0.0095)\mathrm{ps}$ and $BR_\tau(D_s^+) = 0.07 \pm 0.04$. This leads to a value $(\tau(D^+)/\tau(D_s^+))(1 - BR_\tau(D_s^+))_{exp} = 1.971 \pm 0.096$. Comparison of these two results shows that they are consistent within their errors. A relatively large error of the result obtained through relation (22) originates to a great extent from the expression (20) where the inclusive semileptonic decay rate of D_s^+ is expressed in terms of respective quantities for the other two charmed mesons. In this relation a large factor c^2/s^2 multiplies a small quantity $\Gamma_{SL}(D^+) - \Gamma_{SL}(D^0)$ (the inclusive decay rates for these two charmed mesons are numerically very close). In the final expression, this fact contributes very little to the central value, but gives a significant contribution to the error since $\Gamma_{SL}(D^+)$ and $\Gamma_{SL}(D^0)$ are treated as independent quantities and their individual errors are significantly larger than their difference. The consequences of these facts can be better observed if one performs the following analysis. One can take that $\Gamma_{SL}(D^+)$ and $\Gamma_{SL}(D^0)$ are identically equal. This approximation removes the problematic term of a large factor multiplying a small quantity. This procedure changes the central value at the permille level while the error is almost halved (even with this reduced errors our two results are in a 2σ interval).

The procedures presented so far are by no means restricted to the calculation of the lifetimes of Ξ_c^+ and D_s^+. Any inclusive quantity (such as semileptonic branching ratios) for these hadrons can be expressed by means of inclusive quantities of the other two charmed hadrons belonging to the same multiplet. Similar relations can also be established in multiplets of b hadrons bearing in mind that, e.g., phase-space corrections in the b case can be substantial. Nevertheless, the full success of this approach is dependent on accumulation of experimental data [6] and measurement of inclusive decay rates of suppressed decay modes.

Considerations displayed in this paper are motivated by recent experimental results on charmed baryon lifetimes and the need to establish whether a standard existing formalism can be brought into agreement with these results by eliminating or reducing some of its uncertainties. Our formalism procures model-inedependent results with the assumption of some symmetries. Apart from these desirable properties, the theoretical appeal of our approach consists in expressing some measurable quantity of a heavy hadron in terms of measurable quantities of other heavy hadrons from the same $SU(3)_{flavor}$ multiplet. This feature enables us to set a new course in testing the formalism of inclusive weak decays. Using relations such as (12) and (22) one can use the data for those hadrons the decays of which are more amenable to experimental determination to produce predictions for hadrons where experimental data lack or need theoretical verification (like in the Ξ_c^+ case). As any advantage, this one has its price, too. One has to introduce inclusive decay rates of singly Cabibbo suppressed modes which so far have not been measured (as inclusive modes). Use of data on exclusive decay modes can give a reasonable estimate of necessary decay rates. Nevertheless, the full strength of our approach would manifest itself if direct measurements of inclusive decay rates of singly Cabibbo suppressed modes of Λ_c^+ should be possible in the near future. Even better and more detailed data on exclusive decay modes of Λ_c^+ could improve our understanding of new experimental data on the Ξ_c^+ lifetime.

The real challenge now faces the experimental community. There is a clear indication that by measuring the parameters of one heavy hadron (Λ_c^+) we can draw definite conclusions on the other heavy hadron (Ξ_c^+). These conclusions may clarify the question of applicability of the *HQE* in charmed decays or at least decide whether Ξ_c^+ really fits into the, so far successful, description of charmed baryon lifetime hierarchy.

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